

NEUMANN PROBLEM FOR AN ORDINARY SECOND-ORDER DIFFERENTIAL EQUATION WITH A DISTRIBUTED DIFFERENTIATION OPERATOR

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In this paper, we study a linear ordinary second-order differential equation with a continuously distributed differentiation operator and study a two-point boundary-value problem by the Green function method. Fractional differentiation is presented in the sense of the Riemann-Liouville. Green function of the Neumann problem is constructed in term of a special function. The main properties for Green functions are proved. The explicit form of the solution for two-point boundary value problem to the equation under consideration is defined, when the solvability condition is satisfied. Requirements for the kernel of a continuously distributed differentiation operator that guarantee the fulfillment of the solvability condition for the Neumann problem are indicated.

Keywords: Neumann problem, Green's function, operator of continuously distributed differentiation, operator of fractional Riemann-Liouville differentiation.

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