INVESTIGATION OF FORCED OSCILLATIONS OF A DUFFING OSCILLATOR WITH A VARIABLE FRACTIONAL ORDER DERIVATIVE

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A mathematical model of a Duffing type oscillator with a variable fractional derivative of the Riemann-Liouville derivative is studied. Using the harmonic balance method, algorithms for constructing amplitude-phase characteristics were found. The amplitude-frequency and phase-frequency characteristics were built. The inverse dependence of the Q factor on the order of the fractional derivative is shown. The Q-factor surfaces are constructed depending on the frequency and amplitude.

REFERENCES

1. A. Syta et al. Chaotic vibrations of the Duffing system with fractional damping // Chaos: An Interdisciplinary Journal of Nonlinear Science. 2014. vol. 24, no. 1. Pp. 10-16.

2. Kuznetsov S.P. *Dinamicheskiy khaos* [Dynamic chaos]. M.: Fizmatlit, 2001. 296 p.

3. Uchaikin V.V. *Metod drobnykh proizvodnykh* [The fractional derivative method]. Ulyanovsk: Artichoke, 2008. 512 p.

4. Kim V.A. Duffing oscillator with external harmonic action and variable fractional Riemann-Liouville derivative characterizing viscous friction // Bulletin KRASEC. Physical and Mathematical Sciences. 2016. Vol. 13, No. 2. Pp. 46-49.

5. Parovik R.I. *Khaoticheskiye i regulyarnyye rezhimy drobnykh ostsillyatorov: monografiya* [Chaotic and regular modes of fractional oscillators: monograph] / R.I. Parovik. PretropavlovskKamchatsky: KamSU named after Vitus Bering. 2019. 132 p.

6. Parovik R.I. *Matematicheskoye modelirovaniye nelokal'noy kolebatel'noy sistemy Duffinga s fraktal'nym treniyem* [Mathematical modeling of a nonlocal vibrational Duffing system with fractal friction] // *Vestnik KRAUNTS. Fiziko-matematicheskiye nauki* [KRAUNTS Bulletin. Physics and mathematics]. 2015. No. 1 (10). Pp. 18-24.

7. Kim V.A. *Model'noye uravneniye ereditarnogo ostsillyatora Duffinga s proizvodnoy peremennogo drobnogo poryadka Rimana-Liuvillya, kharakterizuyushchey vyazkoye treniye* [Model equation of the hereditary Duffing oscillator with a derivative of variable fractional

order of Riemann-Liouville characterizing viscous friction] // *Novyy universitet. Tekhnicheskiye nauki* [New University. Technical science]. 2016. No. 4–5 (50–51). Pp. 28-31.

8.   Kim V.A. *Matematicheskaya model' nelineynogo ostsillyatora Duffinga s pamyat'yu* [A mathematical model of a nonlinear Duffing oscillator with memory] // *Aktual'nyye problemy differentsial'nykh uravneniy i ikh prilozheniya: materialy Respublikanskoy nauchn. konf. s uchastiyem zarubezh. uchenykh g. Tashkent* [Actual problems of differential equations and their applications: materials of the Republican scientific. conf. with the participation of abroad. scientists Tashkent]. December 15-17, 2017 Tashkent, 2017. Pp. 253-254.

9.   Kim V.A. *Matematicheskoye modelirovaniye ostsillyatora Duffinga s proizvodnoy peremennogo drobnogo poryadka* [Mathematical modeling of the Duffing oscillator with a derivative of variable fractional order] // *Matematicheskoye modelirovaniye i krayevyye zadachi: m-ly XI vserossiyskoy nauchnoy konferentsii s mezhdunarodnym uchastiyem* [Mathematical modeling and boundary value problems: fields of the XI All-Russian scientific conference with international participation]. T. 2, Samara, May 27-29, 2019 Samara: SamSTU, 2019. Pp. 44-46.

10. Kim V.A. *Matematicheskoye modelirovaniye ostsillyatora Duffinga s proizvodnoy peremennogo drobnogo poryadka* [Mathematical modeling of the Duffing oscillator with a derivative of variable fractional order] // *Nelokal'nyye krayevyye zadachi i rodstvennyye problemy matematicheskoy biologii, informatiki i fiziki: materialy V mezhdunarod. nauchn. konf. k 80-letiyu Adama Maremovicha Nakhusheva, g. Nal'chik, Kabardino-Balkarskaya respublika, 4–7 dekabrya 2018 g* [Nonlocal boundary value problems and related problems of mathematical biology, computer science, and physics: materials of the V international. scientific conf. on the occasion of the 80th anniversary of Adam Maremovich Nakhushev, Nalchik, KabardinoBalkarian Republic, December 4–7, 2018]. Nalchik: Elbrus, 2018. Pp. 103-104.

11. Kim V.A., Parovik R.I. *Raschet maksimal'nykh pokazateley Lyapunova dlya kolebatel'noy sistemy Duffinga so stepennoy pamyat'yu* [Calculation of the maximum Lyapunov exponents for a Duffing oscillatory system with power-law memory] // *Vestnik KRAUNTS. Fiziko-matematicheskiye nauki* [KRAUNTS Bulletin. Physics and mathematics]. 2018. No. 3 (23). Pp. 98-105.

12. Kim V.A. *Khaoticheskiye rezhimy ostsillyatora Duffinga s proizvodnoy peremennogo drobnogo poryadka Rimana-Liuvillya* [Chaotic modes of the Duffing oscillator with a derivative of variable fractional order of Riemann-Liouville] // *Aktual'nyye problemy prikladnoy matematiki: materialy IV mezhdunarod. nauchn. konf., p. Terskol, Kabardino-Balkarskaya respublika, 22–26 maya 2018 g* [Actual problems of applied mathematics: materials of IV international. scientific conf., Terskol settlement, Kabardino-Balkarian Republic, May 22–26, 2018]. Nalchik: Elbrus, 2018. Pp. 121-122.

13. Malkin I.G. *Metody Lyapunova i Puankare v teorii nelineynykh kolebaniy* [Lyapunov and Poincare methods in the theory of nonlinear oscillations]. M.: Publ. 2, 2004. 248 p.

14. Nakhushev A.M. *Drobnoye ischisleniye i yego primeneniye* [Fractional calculus and its application]. M.: Fizmatlit, 2003. 272 p.

15. Oseledets V.I. *Mul'tiplikativnaya ergodicheskaya teorema. Kharakteristicheskiye pokazateli Lyapunova dinamicheskikh sistem* [Multiplicative ergodic theorem. Characteristic Lyapunov exponents of dynamical systems] // Tr. Moscow Mathematical Society. 1968. Vol. 19. Pp. 197-231.

16. Parovik R.I. *Konechno-raznostnyye skhemy dlya fraktal'nogo ostsillyatora s peremennymi drobnymi poryadkami* [Finite-difference schemes for a fractal oscillator with variable fractional orders] // *Vestnik KRAUNTS. Fiziko-matematicheskiye nauki* [KRAUNTS Bulletin. Physics and mathematics]. 2015.Vol. 11. No. 2. Pp. 88-95.

17. Parovik R.I. *Ob issledovanii ustoychivosti ereditarnogo ostsillyatora Van-der-Polya* [On the study of the stability of the hereditary oscillator of Van der Pol] // *Fundamental'nyye issledovaniya* [Fundamental research]. 2016. No. 3 (2). Pp. 283-287.

18. Petukhov A.A., Reviznikov D.L. *Algoritmy chislennykh resheniy drobno-differentsial'nykh uravneniy* [Algorithms for numerical solutions of fractional differential equations] // *Vestnik MAI* [Bulletin of the Moscow Aviation Institute]. 2009.V. 16. No. 6. Pp. 228-243.

19. Gao X., Yu J. Chaos in the fractional order periodically forced complex Duffing's oscillators // Chaos, Solitons & Fractals. 2005. Vol. 24, № 4. Pp. 97-104.

20. Petras I. Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation. New York: Springer, 2010. 180 p.

21. Volterra V. Leçons sur les fonctions de lignes, professées a la Sorbonne en 1912. Paris: Gauthier-Villars, 1913. 240 p.